Journal of Advance Research in Science and Social Science (JARSSC)
Official Publication of Indian Mental Health & Research Centre
ISSN: 2582-2004
Volume 01, Issue 01
DOI: 10.46523/jarssc.01.01.23
Multidisciplinary, Open Access
Impact Factor: 3.575
Crossref DOI Prefix 10.46523

Motion for the Multiphase Flow inside the Ground Water

Dr. Rajiva Dixit
Department of Mathematics, B. S. N. V. P. G. College, Lucknow, U. P., India
Email: rajivadixit12@gmail.com
mailto:pramod.pkpant96@gmail.com

ABSTRACT

The flow of single-phase fluid in a pore area is best illustrated by Darcy's law. In the petroleum industry and in other technological applications, transportation is modeled by submitting a multiphase generalization of Darcy law. In these interactions, different pressures are defined in each phase which includes differences known as capillary pressures, determined by surface tension, micro pore geometry and surface chemistry of the solid surface. With flow rates, relative access is defined which associates the flow rate of each fluid volume with its pressure gradient. In the present paper, there are excerpts and analyzes about the equation for the distribution of groundwater flow and some important results have been established. The effect of liquid on the flow rate is calculated according to the viscosity term. Pressure pulses listen to the distribution equation and not the wave equation. Then they move at a slower pace than they do at a steady pace. The results shown in this paper are very useful in the science of the world and in the petroleum industry.

Keywords: Darcy’s law, Diffusion equation, Ground water, Multiphase flow.

1. Introduction

Groundwater systems contain many of the world's fresh water, which provides much of the water needed for human society and ecosystems. Underground water is usually from hundreds to thousands of years, making it the most independent source of water in the caprice of the season associated with many surface water sources. Due to the world's largest source of drinking water, protection of this source is essential to ensure widespread access to reliable sources of clean water. Conditions of groundwater pollution are common, and many can be identified as serious risks to public health. But from long-term habitats it often extends to groundwater pollution and pollution may be involved with long-term harmful impacts on polluted resources. Non-aqueous phase liquids (NAPLs) represent a contamination phase where existing remedial strategies are particularly prevalent. NAPL pollution systems are common, resulting in improper disposal of solvents used in the industry, leakage of underground storage tanks containing petroleum products, disposal and refining products and the inclusion of coal gas [1], [4]. NAPLs do not mix in water, and many melt in small amounts. Once NAPLs are introduced into the system the

Received: 15.07.2018 **Accepted:** 07.08.2018 **Published**: 07.08.2018

pollution can continue for decades or even hundreds of years. The development of useful corrective strategies for these systems has not been very successful, and the standard mathematical methods used to provide flow behavior of these systems are subject to many potential problems, which greatly reduce their predictive capacity [5]. The modeling methods used fail to adequately respond to the fluid levels, and more accurate mathematical interpretations are needed to analyze the risks involved in corruption, improve basic understanding of system behavior, and develop corrective strategies related to these systems.

Consideration of perforated media within a multi-dimensional framework emerges an emerging concept that exploits a mature perception that operates on a small scale in length as a means of interpreting large-scale schemes. Many body systems can be associated with a series of height measurements; each is associated with a specific mathematical formula that describes the behavior of the system on that scale. Multi-scaled structures provide relationships between these different meanings, which provide a series of mathematical formations.

When this is used in small media the method can be used to bind large thermodynamic forms and storage figures to those operating in the port scale, otherwise known as small scale. This is useful if the macroscopic closure relationship is unreliable or incomplete; minor closure relationships are often better known. Microscopic simulation can therefore be used to provide information on extreme behavior, judges to make guesses easier, and to produce appropriate relationships for greater closure. These studies rely heavily on computational methods to provide real-world solutions for microscopic analysis of porous medium flow. Computer analysis provides opportunities to incorporate larger and more realistic behaviors of small analytical behavior into larger model analysis.

In Germany and many other countries more than half of the population depends on groundwater as a source of drinking water [6]. Problems with groundwater quality arise from landfills, leaking tanks and accidental spills of industrial equipment. Removing these substances from the underground space is very complex and difficult, if possible [7]. In order to design effective remedies it is important to understand the regulatory mechanisms of flow and transport in porous sources. Mathematical modeling is one of the most important ways to help get targeted. Incorporating a wide range of physical and geometric details in mathematical models that require the use of numerical algorithms and compatible computers on a large scale, both are a major problem in this analysis. Among the most common toxic substances that threaten groundwater quality are so-called NAPL liquids such as petroleum products, chlorinated hydrocarbons etc. These chemicals are low in soluble in water and should be treated as separate components in the groundwater.

Received: 15.07.2018 **Accepted:** 07.08.2018 **Published**: 07.08.2018

Now, these objects contain flexible components that are transported in the air phase. When the supply of LNAPL stops, a certain amount of it remains immovable in the soil at the base of the residue. The density of NAPL (DNAPL) is heavier than water and its flow behavior in a waterless area is the same. But because of its great congestion it goes down again with full sinners. But due to the effects of capillary heterogeneities in the soil it plays an important role in the flow of multiphase. The regions of the small holes are not filled with liquid until the accumulation of vital fluids accumulates. The dimensions of these regions may vary from centimeters leading to the extraordinary spread of NAPL on the sides to meters in the construction of DNAPL pools. NAPLs provide a long-term problem with groundwater quality. Initial entry may occur in hours or days while the resolution process may occur over the years. NAPL's low concentration of system 10 makes water unsuitable for drinking. With the large number of procedures listed it is evident that the mathematical modeling of the correction processes can be very difficult. In the simplest example of an unchanging two-phase flow, the mathematical model is related to two separate non-linear calculated segments based on time. But the detailed geometry of the medium porous is impossible to determine its complex structure is effectively known for several parameters in mathematical calculations. It is a basic problem for all medium flow models to get these parameters. But due to the heterogeneity of the porous medium at different lengths these functional parameters depend on the scale. Different ways have been found to address this problem. Here we have talked about stochastic modeling [8], upscaling [2] and parameter identification [6] .We have so far focused on the problems of groundwater processing as our motivation for the consideration of excess fluid flow in porous sources. In addition there are other important applications of these types such as oil tank exploitation and safety analysis of underground waste storage facilities. The latter system is often complicated by the presence of cracks in the solid rock [3].

With the flow of porous media, Darcy's equation has been used. Darcy's calculations are generally based on the principle of linear regression between speed and compression grade in narrow media. The line element is expressed as porosity and represents the flow resistance in solid media. The flow process in pore media is governed by certain visual conditions such as viscous forces and forces arising from the solidity of the surface between solids and liquids, but also more tensions between different layers of liquid. The flow process is involved in a goal developed by the use of the equation, but it takes more effort to simulate resolving the force equation than using Darcy's equation. For this reason Darcy's figure is widely used to mimic fluid flow through pore media.

Received: 15.07.2018 **Accepted:** 07.08.2018 **Published**: 07.08.2018

2. Governing Equations and Boundary Conditions

The law governing the flow of fluids through porous media is Darcy's law, which was given by the Henry Darcy in 1856 on the basis of his experiments on vertical water filtration through sand beds. The detailed derivation related to diffusion was given by Zimmerman R. W. in 2002 [9]. The Darcy found that his data could be described by the equation

$$Q = \frac{C\,A\,\Delta(P - \rho g z)}{L} \tag{1}$$

The terms used above are defined as:
P = pressure [Pa],
ρ = density [kg/m^3],
g = gravitational acceleration [m/s^2],
z = vertical coordinate (measured downwards) [m],
L = length of sample [m],
Q = volumetric flow rate [m^3/s],
C = constant of proportionality [$m^2/Pa\ s$],
A = cross-sectional area of sample [m^2]

Any consistent set of units can be used in Darcy's law, such as SI units, C.G.S. units etc. But in the oil industry it is common to use "oilfield units", that are inconsistent. The Darcy's law is mathematically same as other linear transport laws, such as Ohm's law for electrical conduction, Fick's law for solute diffusion and Fourier's law for heat conduction. By the fluid mechanics we know that Bernoulli's equation contains the terms:

$$\frac{P}{\rho} - g\,z + \frac{V^2}{2} = \frac{1}{\rho}\left(P - \rho\,g\,z + \rho\frac{v^2}{2}\right) \tag{2}$$

The term P/ρ is related to the enthalpy per unit mass, $g\ z$ is the gravitational energy per unit mass, $v^2/2$ is the kinetic energy per unit mass. But the fluid velocities in a reservoir are small then the third term can be neglected. And we see that the term $(P - \rho g\ z)$ represents a term of energy type. That seems reasonable that the fluid can flow from regions of higher to lower energy so, the driving force for flow must be the gradient of $(P - \rho g\ z)$. But due to Darcy's analysis, it has been found that all other factors being equal, Q is inversely proportional to the fluid viscosity. It is therefore suitable to factor out μ, and put $C = k/\mu$, where k is known as the permeability. On taking the volumetric flow per unit area, $q = Q/A$. Now the Darcy's law can be written as:

Received: 15.07.2018 **Accepted:** 07.08.2018 **Published**: 07.08.2018

$$q = \frac{Q}{A} = \frac{k}{\mu}\frac{\Delta(P - \rho g z)}{L} \qquad (3)$$

The flux q has the dimension [m/s]. It is perhaps easier to say of these units as [m^3/m^2s].

For transient processes in which the flux varies from the point to point, we can write a differential form of Darcy's law. In the vertical direction, this equation can be written as:

$$q_v = \frac{Q}{A} = -\frac{k}{\mu}\frac{d(P - \rho g z)}{dz} \qquad (4)$$

The suffix v is taken for vertical flow. The minus sign is taken since the fluid flows in the direction from higher potential to lower potential. The differential form of Darcy's law for one-dimensional horizontal flow can be

$$q_h = \frac{Q}{A} = -\frac{k}{\mu}\frac{d(P - \rho g z)}{dx} = -\frac{k}{\mu}\frac{dP}{dz} \qquad (5)$$

The suffix v is taken for vertical flow. In most rocks the permeability k_h in the horizontal plane is different than the vertical permeability, k_v ; in most of the cases, $k_h > k_v$. The permeabilities in any two orthogonal directions within the horizontal plane have the difference. So, in this course we shall generally take: $k_h = k_v$. The permeability is the function of rock type, that varies with stress, temperature etc., but it does not depend on the fluid; the effect of the fluid on the flow rate is accounted for by the term of viscosity in the above equations. The permeability has units of m^2, but in mathematical use it is conventional to use "Darcy" units, defined as:

$$1 Darcy = 0.987 \times 10^{-12}\ \mathrm{m}^2 \approx 10^{-12}\ m^2$$

The Darcy unit is defined such that a rock having a permeability of 1 *Darcy* would transmit 1 c.c. of water with viscosity 1 *cP per second*, through a region of 1 sq. cm. cross-sectional area, if the pressure drop along the direction of flow were 1 *atm per cm.* Many soils and sands that mathematicians must deal with have permeabilities on the order of a few *Darcies*. The original purpose of the "*Darcy*" definition was thus to avoid the need for using small prefixes such as 10^{-12} etc. But a Darcy is nearly a round number in SI units, so conversion between the two is easy. The numerical value of k for a given rock depends on the size of the pores in the rock, d as well as on the degree of interconnectivity of the void space. So that

$$k \approx d^2/1000$$

Received: 15.07.2018 **Accepted:** 07.08.2018 **Published**: 07.08.2018

The term *d* denotes the diameter of the pores. The permeabilities of different types of rocks and soils vary over many orders of magnitude. However, the permeabilities of petroleum reservoir rocks tend to be in the range of 0.001-1.0 *Darcies*. So it is convenient to refer the permeability of reservoir rocks in units of "*milliDarcies*" (*mD*), which equal 0.001 *Darcies*. In some reservoirs, the permeability is due mainly to an interconnected network of fractures. The permeabilities of fractured rocks tend to be in the range 1 *mD* to 10 *Darcies*. In a fractured reservoir, the reservoir scale permeability is not closely related to the core scale permeability that anyone can measure it.

If the fluid is in static equilibrium then $q = 0$, so the differential equation will be

$$\frac{d(P - \rho gz)}{dx} = 0 \Rightarrow P - \rho gz = \text{constant} \tag{6}$$

If we take $z = 0$ i.e., at sea level, where the fluid pressure is equal to the atmospheric pressure, then we have

$$P_s = P_a + \rho\, g\, z \tag{7}$$

Here P_s, P_a represent the static pressure and atmospheric pressure respectively. But we always measure the pressure above the atmospheric pressure, so we can neglect the term P_a in the above equation (7). We can see by comparing equation (7) with equation (4) that only the pressure above and beyond the static pressure given by equation (2) plays a role in driving the flow. So the term ρgz is useless, as it only contributes to the static pressure and but does not contribute to the driving force for the flow. Then after removing the term, the equation for correct pressure will be

$$P_c = P - \rho gz \tag{8}$$

Now the Darcy's law in the terms of corrected pressure for the horizontal flow can be written as:

$$q = \frac{Q}{A} = -\frac{k}{\mu}\frac{dP_c}{dx} \tag{9}$$

Instead of using sea level i.e., $z = 0$, we can take $z=z_0$ as a datum i.e., the amounts of initial oil in place lie above and below $z=z_o$. So we get

$$P_c = P - \rho\, g\, (z - z_o) \tag{10}$$

Received: 15.07.2018 **Accepted:** 07.08.2018 **Published**: 07.08.2018

The data rate selection is intangible, in the sense that it provides a constant time at adjusted pressure so as not to contribute to the increase in pressure. The pressure Pc described in equation (10) can be interpreted as the pressure of the analytical fluid at z-zo depth which will be equal to the liquid and present at the actual pressure at the z-depth. Darcy's law should be a macroscopic law aimed at targeting regions larger than the size of a single hole. Now we can talk about entering a place in the water reservoir, we will not be referring to the entrance to a limited area of mathematics as a given point can be a grain of sand and not a hole. Accessibility property is actually defined as a patchwork area, not a single patch. Accessibility is therefore a descriptive element of measurement in a particular area around a point (x, y, z). Now the region should be large enough to cover a large number of pores. The pressure P used in Darcy's law is actually an intermediate pressure taken from a small area of space.

Darcy's law itself does not provide sufficient information for time-dependent solutions i.e., temporary problems involving groundwater flow. So in order to get the perfect governing body that works on these problems, we will first have to find the mathematical expression of the weight loss goal. Maintaining mass equation means the balance between the rate of weight change by the unauthorized volume and the weight gain in the boundary area. In its full form, this can be expressed as follows:

$$\frac{\partial}{\partial t}\iiint \rho\,\phi\,dV + \iint \rho\,\bar{v}.\bar{n}\,dS = \iiint q\,dV \qquad (11)$$

In this equation the double and triple integrals are taken over surface and volume respectively and $\bar{v}, \bar{n}, \rho, q$ and ϕ represent the velocity vector, unit normal vector, fluid density, external mass flow rate and porosity respectively. The right hand side term of the equation (11) can be changed into a volume integral form by the using the Gauss' divergence theorem as:

$$\iint \rho\,\bar{v}.\bar{n}\,dS = \iiint \nabla.(\rho\,\bar{v})\,dV \qquad (12)$$

So, for a fixed control volume, the integral form of the conservation law will be

$$\iiint \left[\frac{\partial(\rho\,\phi)}{\partial t} + \nabla.(\rho\,\bar{v}) - q\right] dV \qquad (13)$$

Now the differential form the conservation equation for mass can be written in coordinate invariant form as:

Received: 15.07.2018 **Accepted:** 07.08.2018 **Published**: 07.08.2018

$$\frac{\partial(\rho\,\phi)}{\partial t}+\nabla.(\rho\,\bar{v}) = q \qquad (14)$$

For the multiphase flow, it is necessary to account for the saturation of each phase. So that the equation (14) within each phase α can be written as:

$$\frac{\partial(\rho_\alpha\, S_\alpha \phi)}{\partial t}+\nabla.(\rho_\alpha\,\bar{v}_\alpha) = q_\alpha \qquad (15)$$

3. Discussion and Results

The transient flow of fluid in a pore space is controlled by a type of differential component known as the diffusion equation. Detailed discovery and distribution-related discussion was provided by R. W. Zimmerman in 2002 [14] that is, discussed in this paper. So in order to get the calculation, we combine Darcy's law with the law of mass conservation and the figure that describes the process by which the liquid is stored inside a perforated stone. Now we use the product function division (ρ ϕ), we find

$$D(\rho\,\phi) = \rho\, D\phi + \phi\, D\rho, \qquad (16)$$

$$D(\rho\,\phi) \;= \phi\left[\left(\frac{1}{\phi}\frac{d\phi}{dP}\right)\frac{dP}{dt} + \rho\left(\frac{1}{\rho}\frac{d\rho}{dP}\right)\frac{dP}{dt}\right]$$

$$D(\rho\,\phi) \;= \rho\,\phi\,\left(c_R + c_f\right) D(P) \qquad (17)$$

The terms c_R , c_f are the compressibility of the rock and the fluid respectively.

The equation for mass conservation for the fluid flow is taken as:

$$-\frac{d(\rho\,q)}{dx} = \frac{d(\rho\,\phi)}{dt} \qquad (18)$$

By using the Darcy's law, we have

$$-\frac{d(\rho\, q)}{dx} = \frac{k\rho}{\mu}\left[\frac{d^2\, P}{dx^2} + c_f\left(\frac{dP}{dx}\right)^2\right] \qquad (19)$$

From the equations (17) and (19), we get

$$\frac{d^2\, P}{dx^2} + c_f\left(\frac{dP}{dx}\right)^2 = \frac{1}{K}\rho\,\phi\,\left(c_R + c_f\right) D(P) \qquad (20)$$

But we know that,

$$\frac{d^2\, P}{dx^2} \gg c_f\left(\frac{dP}{dx}\right)^2$$

On neglecting the term $c_f\left(\frac{dP}{dx}\right)^2$ $, i.e.$

$$c_f\left(\frac{dP}{dx}\right)^2 \approx c_f\left[\frac{\mu\, Q}{2\,\pi\, k\, H\, R}\right]^2 \qquad (21)$$

$$\frac{d^2\, P}{dx^2} \approx \frac{\mu\, Q}{2\,\pi\, k\, H\, R^2} \qquad (22)$$

So we can neglect the nonlinear term in equation (20), we get the diffusion equation as given

$$\frac{dP}{dt} = \frac{K}{\phi\,\mu\,(c_\phi + c_f)}\frac{d^2P}{dx^2} \qquad (23)$$

Here $\left(c_\phi + c_f\right) = c$ is the total compressibility.

The parameter which governs the rate at which fluid pressure diffuses through the rock is the hydraulic diffusivity D_H , that is defined by

$$D_H = \frac{k}{\phi\, \mu\, (c_\phi + c_f)} \tag{24}$$

The distance *d* at which a pressure disturbance will travel during an elapsed time *t* is given as

$$d = \sqrt{4\, D_H t} \tag{25}$$

For the multi-phase flow if we have assume that the pores of the rock are filled with two components, oil and water, and often also contain some hydrocarbons in the gaseous phase. So we have to find the governing flow equations for an oil and water system, in the general form. From the rock properties module that Darcy's law can be generalized for two-phase flow by including a relative permeability factor for each phase, we have

$$q_w = \frac{-kk_{rw}}{\mu_w}\frac{dP_w}{dx} \tag{26}$$

$$q_o = \frac{-kk_{ro}}{\mu_o}\frac{dP_o}{dx} \tag{27}$$

Here the subscripts *w* and *o* are used for oil and water respectively. The two relative permeability functions are supposed to be known functions of the phase saturations. For the oil-water system, the two saturations are necessarily related to each other by the relationship

$$S_w + S_o = 1 \tag{28}$$

The pressures in the two phases at every point in the reservoir must be different. If the reservoir is oil-wet then the two pressures will be given by

$$P_o - P_w = P_c\, S_o \tag{29}$$

Here the capillary pressure P_c is given by the rock-dependent function of oil saturation.

But the volume of the oil in a given region is equal to the total pore volume multiplied by the oil saturation then by the equations of the conservation of mass for the two phases can be written by inserting a saturation factor in the storage term as given

$$-\frac{d(\rho_0 q_0)}{dx} = \frac{d(\phi\rho_0 S_0)}{dt} \tag{30}$$

$$-\frac{d(\rho_w w)}{dx} = \frac{d(\phi\rho_w S_w)}{dt} \tag{31}$$

The densities of the two phases are related to their respective phase pressures with the equation of state as given by

$$\rho_0 = \rho_0(P_0) \tag{32}$$

Received: 15.07.2018 **Accepted:** 07.08.2018 **Published**: 07.08.2018

$$\rho_w = \rho_w(P_w) \tag{33}$$

Here the temperature is taken as constant.

Now, the porosity must be the function of the phase pressures *Po* and *Pw*. The above two pressures independently affect the porosity. Now, the capillary pressure P_c is generally small so that

$$Po \approx Pw$$

From which we can use the pressure-porosity relationship that would be obtained under single-phase conditions, *i.e.,*

$$\phi = \phi\,(P_0) \tag{34}$$

If the fluid is taken as slightly compressible or if the pressure variations are small then the equations of state are written as

$$\rho(P_o) = \rho_{0i}[1 + c_o\,(P_o - P_{oi}\,)] \tag{35}$$

Here the subscript "i" is used for the initial state, and the compressibility c_o is taken as a constant.

The distance *d* at which a pressure disturbance will travel during an elapsed time *t* is given as

$$d = \sqrt{4\,D_H t}$$

The time required for a pressure disturbance to travel a distance *d* is found by

$$t = \frac{\phi\,\mu\,c\,d^2}{4k}$$

The pressure pulses obey a diffusion equation not a wave equation. So, they travel at a speed that continually decreases with time rather than travelling at a constant speed.

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